Constraint programming and (dashed) string solving

Roberto Amadini

Department of Computer Science and Engineering, University of Bologna, Italy

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String analysis

- Interest in string analysis active and growing over last decade
  - Test-Case Generation
  - Program Analysis
  - Model Checking
  - Web Security
- Static string analysis: over-approximation of string computations
  - E.g., Abstract Interpretation with string abstract domains
- Dynamic string analysis: under-approximation of string computations
  - E.g., (Dynamic) Symbolic Execution (DSE) of programs with strings
    - We need (string) constraint solving to solve path condition constraints

String constraint solving

- String constraint solving (SCS) = solving combinatorial problems with string variables and constraints on given alphabet Σ
  - Foundations lay in theory of automata and combinatorics on words
  - Word equations $L = R$ with $L, R \in (\Sigma \cup \mathit{Vars})^*$ are central in SCS
  - General FOL theory undecidable, quantifier-free theory decidable (Makanin, 1977)

- We can classify string variables into:
  - fixed-length: given $\lambda \in \mathbb{N}$, only take values in $\{w \in \Sigma^* \mid |w| = \lambda\}$
  - bounded-length: given $\lambda \in \mathbb{N}$, only take values in $\{w \in \Sigma^* \mid |w| \leq \lambda\}$
  - unbounded-length: they can take any value in Σ^*

- String constraints: length, concatenation, regular, find/replace(-all), lexicographic ordering, conversion string $\leftrightarrow$ numbers

String constraint solvers

- We can classify string solvers into:
  - automata-based: string variables represented by automata, string constraints mapped to automata operations
  - word-based: natively handle theory of word-equations + extensions
  - unfolding-based: reduce to sequences of $k \geq 0$ variables of type T
  - No sharp distinction, efficient SOTA solvers are hybrid

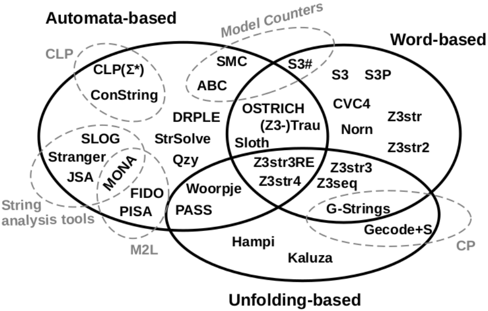


Figure from [Amadini, 2021]

String constraint solvers

- Most SOTA "general-purpose" string solvers are SMT-based
  - SMT-LIB developed a theory of unicode strings
- Some Constraint Programming (CP) proposals too
- From CP perspective, SCS = solving CSP/COP with bounded-length string variables: a maximum string length λ is fixed
  - Goal: assigning a consistent string literal to each string variable
- CP proposals are mainly unfolding-based:
  - Bounded-length sequence (B.L.S.) variables [Scott *et al.*, 2017]
    - String variable x unfolded into max $|x|$ integer variables c_i^x = i-th char of x (possibly empty), plus 1 integer variable n_x for $|x|$
  - Dashed string (D.S.) variables [Amadini *et al.*, 2020]

Dashed strings

- Dashed strings: simplified regular expressions representing set of strings in a more compact way w.r.t. B.L.S.
  - Inspired by Bricks abstract domain by [Costantini *et al.*, 2015]

- D.S. enable a more "lazy" unfolding w.r.t. B.L.S.: blocks group together "similar regions" of the target string
  - If no clue on length upper bound, B.L.S. needs $\lambda + 1$ integer variables

- E.g., D.S. $\{b, c\}^{1,1}\{a\}^{0,1}\{d\}^{1,2}$ denotes all the strings:
  - Starting with 1 b's or c's, followed by 0 or 1 a's, followed by 1 or 2 d's
  - . . . i.e., the set of strings $\{bd, bdd, bad, badd, cd, cdd, cad, cadd\}$

- With B.L.S. representation it would be $\{b, c\}\{a, d\}\{\epsilon, d\}\{\epsilon, d\}$, denoting $\{ba, bd, ca, cd, bad, bdd, cad, cdd, badd, bddd, cadd, cddd\}$
  - More blocks, less precise abstraction

Dashed strings

- Formally, a D.S. is a concatenation of blocks $S_1^{l_1,u_1} S_2^{l_2,u_2} \cdots S_k^{l_k,u_k}$ with $S_i \subseteq \Sigma$ and $0 \leq l_i \leq u_i \leq \lambda$
- Each block $S_i^{l_i,u_i}$ denotes $\gamma(S_i^{l_i,u_i}) = \{w \in S_i^* \mid l_i \leq |w| \leq u_i\}$
- If $\mathbb{S} = \{w \in \Sigma^* \mid |w| \leq \lambda\}$, each D.S. $X = S_1^{l_1,u_1} S_2^{l_2,u_2} \cdots S_k^{l_k,u_k}$ denotes: $\gamma(X) = \left(\gamma(S_1^{l_1,u_1}) \cdot \gamma(S_2^{l_2,u_2}) \cdot \cdots \cdot \gamma(S_k^{l_k,u_k})\right) \cap \mathbb{S}$
- E.g, if $\Sigma = \{a, b, c\}$ and $\lambda = 3$, $X = \{a\}^{1,2}\{b, c\}^{0,2}$ denotes $\gamma(X) = \{a, ab, ac, abb, abc, acb, acc, aa, aab, aac\}$
  - $aabb, aabc, aacb, aacc \notin \gamma(X)$ because they have length $> \lambda$

Graphical interpretation

- Each block $S_i^{l_i,u_i}$ can be seen as:
  - a continuous segment of length l_i (the mandatory part of the block), followed by
  - a dashed segment of length $u_i - l_i$ (the optional part of the block)
- E.g., graphical representation of $X = \{\texttt{B,b}\}^{1,1}\{\texttt{o}\}^{2,4}\{\texttt{m}\}^{1,1}\{\texttt{!}\}^{0,3}$

- $\gamma(X) = \{\texttt{Bom}, \texttt{bom}, \texttt{Boom}, \texttt{boom}, \ldots, \texttt{Booom!!!}, \texttt{booom!!!}\}$

Dashed string equation

- Unfortunately, D.S. cannot precisely represent all the $W \subseteq \Sigma^*$

- E.g., if $W = \{ab, ba\}$ there is no a "best representation" X for W s.t. $\gamma(X) = W$
  - We may take $\{a\}^{0,1}\{b\}^{1,1}\{a\}^{0,1}$, $\{b\}^{0,1}\{a\}^{1,1}\{b\}^{0,1}$ or $\{a, b\}^{2,2}$ as over-approximations: $\gamma(X) \supset W$

- We need workarounds to find "good enough" approximations and domains' refinement

- Given D.S. X, Y we define the D.S. equation between X and Y as a refinement operation $\textsc{Equate}(X, Y)$ s.t.
  - if $\textsc{Equate}(X, Y) = \bot$, then $\gamma(X) \cap \gamma(Y) = \emptyset$
  - if $\textsc{Equate}(X, Y) = (X', Y')$, then $\gamma(X') \subseteq \gamma(X)$, $\gamma(Y') \subseteq \gamma(Y)$, and $\gamma(X) \cap \gamma(Y) = \gamma(X') \cap \gamma(Y')$

Dashed string equation

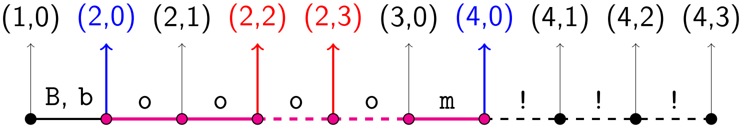
- Most of D.S. propagators we propose are based on sweep-based equation algorithms matching blocks against portions of D.S.

- Suitable PUSH and STRETCH operations are used to find the earliest/latest start/end positions where a block can match a D.S.
    - PUSH: consumes the least characters of a block, can "jump"
    - STRETCH: consumes the most characters of a block
    - ESP...LEP $=$ feasible region $\supseteq$ mandatory region $=$ LSP...EEP
    - If there is no feasible match, $\perp$ is returned

- Matching regions are then used to possibly refine the blocks
    - A normalization is used to remove spurious configurations
    - E.g., $\textsc{Norm}(\{a\}^{0,2}\,\emptyset^{1,5}\,\{a\}^{1,1}) = \{a\}^{1,3}$

Dashed string equation

- E.g., matching $B = \{\texttt{o}, \texttt{m}, \texttt{g}\}^{2,6}$ vs. $X = \{\texttt{B,b}\}^{1,1}\{\texttt{o}\}^{2,4}\{\texttt{m}\}^{1,1}\{\texttt{!}\}^{0,3}$



- The earliest start position of B is $(2, 0)$
- The latest start position of B is $(2, 3)$
- The earliest end position of B is $(2, 2)$
- The latest end position of B is $(4, 0)$

- Feasible region = ESP...LEP = $X[(2, 0), (4, 0)] = \{\texttt{o}\}^{2,4}\{\texttt{m}\}^{1,1}$
- Mandatory region = LSP...EEP = $X[(2, 3), (2, 2)] = \emptyset^{0,0}$

Dashed string equation

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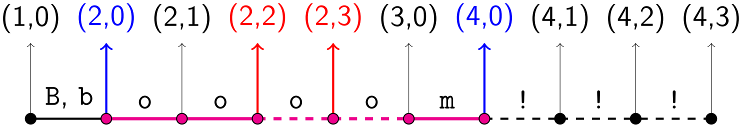


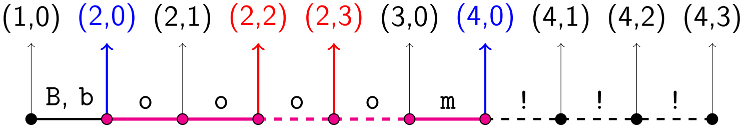
- Feasible region = ESP...LEP = $X[(2,0),(4,0)] = \{\mathtt{o}\}^{2,4}\{\mathtt{m}\}^{1,1}$: B must match X within this region

- Mandatory region = LSP...EEP = $X[(2,3),(2,2)] = \emptyset^{0,0}$: no precise information about which blocks are surely matched by B

- What we know is that ≥ 2 blocks and ≤ 6 blocks of the feasible region must be matched by B

Dashed string equation

- E.g., matching $B = \{\texttt{o}, \texttt{m}, \texttt{g}\}^{2,6}$ vs. $X = \{\texttt{B},\texttt{b}\}^{1,1}\{\texttt{o}\}^{2,4}\{\texttt{m}\}^{1,1}\{\texttt{!}\}^{0,3}$



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- We cannot in general refine B into $\{\texttt{o}\}^{2,4}\{\texttt{m}\}^{1,1}$: there might be blocks before/after B matching $X[(2,0),(4,0)]$

- Surely we can CRUSH the feasible region $\{\texttt{o}\}^{2,4}\{\texttt{m}\}^{1,1}$ into $\{\texttt{o}, \texttt{m}\}^{3,5}$ and refine B into $(\{\texttt{o}, \texttt{m}, \texttt{g}\} \cap \{\texttt{o}, \texttt{m}\})^{2,\min(6,5)} = \{\texttt{o}, \texttt{m}\}^{2,5}$

Dashed string equation

- The worst-case complexity of finding matching positions is linear in the number of blocks of X and Y
  - $\mathsf{ESP}_{k+1} \equiv \mathsf{EEP}_k$, $\mathsf{LSP}_{k+1} \equiv \mathsf{LEP}_k$
  - The refinement might be quadratic, but it's a very rare case
- We proved that our approach is sound
  - Its completeness is still an open issue: if $\gamma(X) \cap \gamma(Y) = \emptyset$, we have no proof that $\textsc{Equate}(X, Y) = \bot$
- We built several propagators for string constraints on top of $\textsc{Equate}$
  - (Dis-)equality, reified equality, concatenation, . . .

Propagators EQUATE-based

- $x = y$
- $x \neq y$
- $b \iff x = y$
- $z = x \cdot y$
- $y = x^n$
- $y = x^{-1}$
- $y = x[i..j]$
- FIND
- REPLACE
- REPLACE-ALL
- $\ldots$

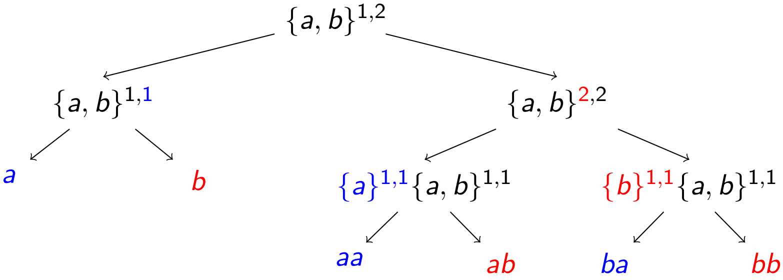
Propagators not EQUATE-based

- $n = |x|$
- $x \prec y$, $x \preceq y$, $x \succeq y$, $x \succ y$
- $x \in \mathcal{L}(R)$
- $i = \mathrm{match}(x, \rho)$

Branching

- Often we need to branch on string variables to get a feasible solution

- We first fix the length of a string variable, then the cardinality of one of its blocks, and finally the base of that block



$\{a, b\}^{1,2}$

$\{a, b\}^{1,1}$

$\{a, b\}^{2,2}$

a

b

$\{a\}^{1,1}\{a, b\}^{1,1}$

$\{b\}^{1,1}\{a, b\}^{1,1}$

aa

ab

ba

bb

Implementation

- We implemented string variables, propagators and branchers into G-STRINGS
  - Experimental extension of GECODE solver, written in C++
- We developed a MiniZinc interface to ease the modeling of SCS problems, and a compiler MiniZinc→SMT-LIB
  - Most state-of-the-art string solvers are SMT-based
- We performed several evaluations over different benchmarks with good results, especially with long strings and big regex
  - E.g., *StringFuzz* benchmarks
- G-STRINGS development a bit quiet now :-(

Conclusions

- We presented a CP approach for SCS based on dashed strings
- Possible extensions
  - New string constraints
  - New branching heuristics
  - Clause learning
  - Portfolios of string solvers
  - . . .
- How to involve students / companies? :)

Thanks for your attention!

References

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